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In ΔABC the following relationship holds:

$$\sum \frac{\left(c+\sqrt{ab}\right)^2}{2(b+c-a)(a+c)} \le \frac{R}{r}+1$$

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$$Known\ result: \sum \frac{1}{s-a} = \frac{4R+r}{sr}$$

$$\left(c+\sqrt{ab}\right)^2 \overset{C-S}{\le} (c+a)(c+b)$$

$$\sum \frac{\left(c+\sqrt{ab}\right)^2}{2(b+c-a)(a+c)} \le \sum \frac{(c+a)(c+b)}{4(s-a)(a+c)} = \frac{1}{4}\sum \frac{c+b}{s-a} = \frac{1}{4}\sum \frac{2s-a}{s-a} =$$

$$= \frac{1}{4}\sum \frac{(s-a)+s}{s-a} = \frac{1}{4}\left(3+s\sum \frac{1}{s-a}\right) = \frac{1}{4}\left(3+s.\frac{4R+r}{sr}\right) = \frac{1}{4}\left(4+\frac{4R}{r}\right) = \frac{R}{r}+1$$

Equality holds for an equilateral triangle.