ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$3\sum a^2 \geq 16F^2 \sum \frac{1}{a^2}$$

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Solution by Tapas Das-India

$$R.H.S = 16F^2 \sum \frac{1}{a^2} \overset{Steining}{\leq} 16r^2s^2 \times \frac{1}{4r^2} = 4s^2$$

$$L.H.S = 3\sum a^2 = 3 \times 2(s^2 - r^2 - 4Rr) = 6\left(s^2 - r(4R + r)\right) \overset{s^2 \geq 3r(4R+r)}{\geq}$$
$$\geq 6\left(s^2 - \frac{s^2}{3}\right) = 4s^2, \ \ so\ L.H.S \geq R.H.S$$

Equality holds for an equilateral triangle.