ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC$ the following relationship holds:

$$\sum_{cyc} \frac{a}{h_a^2} \geq \sum_{cyc} \frac{a}{r_a^2}$$

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$$\sum_{cyc} \frac{a}{h_a^2} = \sum_{cyc} \frac{a^3}{4F^2} = \frac{1}{4F^2} \sum_{cyc} a^3 \quad (LHS)$$

$$\sum_{cyc} \frac{a}{r_a^2} = \sum_{cyc} \frac{a}{\frac{F^2}{(p-a)^2}} = \sum_{cyc} \frac{a(p-a)^2}{F^2} = \frac{1}{F^2} \sum_{cyc} a(p-a)^2 \quad (RHS)$$

$$Let's\ prove\ that\ LHS \overset{?}{\geq} RHS$$

$$\frac{1}{4F^2} \sum_{cyc} a^3 \overset{?}{\geq} \frac{1}{F^2} \sum_{cyc} a(p-a)^2$$

$$\sum_{cyc} a^3 \overset{?}{\geq} \frac{1}{F^2} \sum_{cyc} a(2p-2a)^2$$

$$\sum_{cyc} a^3 \overset{?}{\geq} \frac{1}{F^2} \sum_{cyc} a(b+c-a)^2$$

$$\sum_{cyc} a^3 \overset{?}{\geq} \frac{1}{F^2} \sum_{cyc} a^3 - a^2(b+c) - b^2(a+c) - c^2(a+b) + 6abc$$

$$a^2(b+c) + b^2(a+c) + c^2(a+b) \geq 6abc$$

$$a^2.2\sqrt{bc} + b^2.2\sqrt{ac} + c^2.2\sqrt{ab} \overset{A-G}{\geq} 3\sqrt[3]{8a^2.b^2.c^2.\sqrt{bc}.\sqrt{ac}.\sqrt{ab}} \geq$$

$$6\sqrt[3]{(abc)^3} = 6abc$$

$$Equality\ holds\ for\ a = b = c.$$