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In ΔABC holds :

$$\sum_{cyc} \left(\frac{m_b + m_c}{a} \right) \geq 3^{\frac{n+2}{2}} \left(\frac{2r}{R} \right)^n, \quad n \in \mathbb{N}$$

Proposed by Marin Chirciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \left(\frac{m_b + m_c}{a} \right) &\stackrel{AM-GM}{\geq} \sum_{cyc} \left(\frac{2\sqrt{m_b m_c}}{a} \right)^n \stackrel{AM-GM}{\geq} 3 \left(\frac{8m_a m_b m_c}{abc} \right)^{\frac{n}{3}} \geq \\ &\stackrel{m_a \geq \sqrt{p(p-a)}}{\geq} 3 \left(\frac{8p\sqrt{p(p-a)(p-b)(p-c)}}{abc} \right)^{\frac{n}{3}} = 3 \left(\frac{8pF}{abc} \right)^{\frac{n}{3}} = \\ &= 3 \left(\frac{8pF}{4RF} \right)^{\frac{n}{3}} \stackrel{Mitrinovic}{\geq} 3 \left(\frac{2 \cdot 3\sqrt{3}r}{R} \right)^{\frac{n}{3}} = 3^{\frac{n+2}{2}} \left(\frac{2r}{R} \right)^{\frac{n}{3}} \geq 3^{\frac{n+2}{2}} \left(\frac{2r}{R} \right)^n \end{aligned}$$

Equality holds for $a = b = c$.