ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{2r}{R} \le \frac{w_a w_b w_c}{r_a r_b r_c} \le \frac{R}{2r}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$w_a w_b w_c = \prod \frac{2bc}{b+c} \cos\frac{A}{2} = \frac{8abc}{(a+b)(b+c)(c+a)} \prod \cos\frac{A}{2} = \frac{16Rr^2s^2}{s^2+r^2+2Rr}$$

$$\frac{w_a w_b w_c}{r_a r_b r_c} = \left(\frac{16Rr^2s^2}{s^2+r^2+2Rr}\right) \cdot \frac{1}{s^2 r} = \frac{16Rr}{s^2+r^2+2Rr} \overset{\substack{Mitrinovic \\ Euler}}{\ge}$$

$$\ge \frac{16Rr}{\frac{27}{4}R^2 + \frac{R^2}{4} + R^2} = \frac{16Rr}{8R^2} = \frac{2r}{R}$$

- $$\frac{w_a w_b w_c}{r_a r_b r_c} = \frac{16Rr}{s^2+r^2+2Rr} \overset{\substack{Mitrinovic \\ Euler}}{\le} \frac{16Rr}{27r^2+r^2+4r^2} = \frac{R}{2r}$$

Equality holds for an equilateral triangle.