ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{(b+c)^2}{w_b w_c} \geq 16$$

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Solution by Tapas Das-India

$$w_b w_c \leq \sqrt{s(s-b)}\,.\sqrt{s(s-c)} = s\sqrt{(s-b)(s-c)} \overset{AM-GM}{\leq}$$

$$\leq s.\frac{s-b+s-c}{2} = \frac{s\big(2s-(b+c)\big)}{2} = \frac{as}{2}$$

$$\sum \frac{(b+c)^2}{w_b w_c} \geq \sum \frac{(b+c)^2}{\frac{as}{2}} = \frac{2}{s}\sum \frac{(b+c)^2}{a} \overset{Bergstrom}{\geq}$$

$$\geq \frac{2}{s}\cdot\frac{\big(2(a+b+c)\big)^2}{a+b+c} = \frac{8(a+b+c)}{s} = 8 \times \frac{2s}{s} = 16$$

Equality holds for an equilateral triangle.