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In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} \frac{2r_a^2 - 9r^2}{r_a} \geq 9r$$

Proposed by Marin Chirciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{2r_a^2 - 9r^2}{r_a} &= 2 \sum_{cyc} r_a - 9r^2 \sum_{cyc} \frac{1}{r_a} = 2(4R + r) - 9r^2 \cdot \frac{1}{r} = \\ &= 8R - 7r \stackrel{\text{Euler}}{\geq} 16r - 7r = 9r \end{aligned}$$

Equality holds for : $a = b = c$.