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In ΔABC the following relationship holds:

$$\sum_{cyc}\left(a^2\tan\left(\frac{B}{2}\right).\tan\left(\frac{C}{2}\right)\right)^n\ge 3(4r^2)^n\,,\quad n\in N$$

Proposed by Marin Chirciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$a^2\tan\left(\frac{B}{2}\right).\tan\left(\frac{C}{2}\right)=a^2\frac{\prod_{cyc}\tan\left(\frac{A}{2}\right)}{\tan\left(\frac{A}{2}\right)}=a^2.\frac{r}{s}.\frac{1}{\tan\left(\frac{A}{2}\right)}$$

$$\sum_{cyc}\left(a^2\tan\left(\frac{B}{2}\right).\tan\left(\frac{C}{2}\right)\right)^n=\sum_{cyc}\left(a^2.\frac{r}{s}.\frac{1}{\tan\left(\frac{A}{2}\right)}\right)^n\overset{AM-GM}{\ge}$$

$$\ge 3\left((abc)^2.\left(\frac{r}{s}\right)^3.\frac{1}{\prod\tan\left(\frac{A}{2}\right)}\right)^{\frac{n}{3}}=3\left((4RF)^2.\left(\frac{r}{s}\right)^3.\frac{1}{\left(\frac{r}{s}\right)}\right)^{\frac{n}{3}}=$$

$$=3\left(16R^2F^2.\left(\frac{r}{s}\right)^2\right)^{\frac{n}{3}}=3\left(16R^2.s^2r^2.\left(\frac{r}{s}\right)^2\right)^{\frac{n}{3}}\overset{Euler}{\ge}$$

$$\ge 3(16.4r^2.r^2.r^2)^{\frac{n}{3}}=3(4r^2)^n$$

$$Equality\ \ holds\ \ for: a=b=c$$