In $\triangle ABC$ the following relationship holds:

$$\frac{27r^2}{2R} \leq \sum h_a \sin^2 A \leq \frac{27r}{4}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum h_a \sin^2 A = 2F \sum \frac{\sin A}{a} \cdot \sin A = \frac{F}{R} \sum \sin A = \frac{F}{R} \cdot \frac{s}{R} = \frac{s^2 r}{R^2}$$

$$\sum h_a \sin^2 A = \frac{s^2 r}{R^2} \overset{Mitrinovic}{\leq} \frac{27}{4} R^2 \cdot \frac{r}{R^2} = \frac{27r}{4}$$

$$\sum h_a \sin^2 A = \frac{s^2 r}{R^2} \overset{s^2 \geq \frac{27Rr}{2}}{\geq} \frac{27Rr}{2} \cdot \frac{r}{R^2} = \frac{27r^2}{2R}$$

Equality holds for an equilateral triangle.