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In ΔABC the following relationship holds:

$$\frac{3}{s} \le \sum \frac{AH}{ah_a} \le \frac{3R}{2rs}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \frac{AH}{ah_a} = \sum \frac{2R\cos A}{a\frac{2F}{a}} = \frac{R}{F}\sum \cos A = \frac{R}{r.s}\left(1+\frac{r}{R}\right) \overset{Euler}{\le} \frac{3R}{2rs}$$

$$\sum \frac{AH}{ah_a} = \sum \frac{2R\cos A}{a\frac{2F}{a}} = \frac{R}{F}\sum \cos A = \frac{R}{r.s}\left(1+\frac{r}{R}\right) = \frac{R+r}{r.s} \overset{Euler}{\ge} \frac{2r+r}{r.s} = \frac{3}{s}$$

Equality holds for an equilateral triangle.