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In ΔABC the following relationship holds:

$$\frac{27r}{R} \leq \sum r_a \sin^2 A \leq \frac{27R^2}{16r}$$

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$$Known\ result: \sum \frac{a^2}{s-a} = \frac{4s}{r}(R-r)$$

$$\sum r_a \sin^2 A = \frac{1}{4R^2}\sum a^2.\frac{F}{s-a} = \frac{F}{4R^2}\sum \frac{a^2}{s-a} = \frac{F}{4R^2}.\frac{4s}{r}(R-r) = \frac{s^2}{R^2}(R-r)$$

$$We\ need\ to\ show:$$

$$\frac{s^2}{R^2}(R-r) \leq \frac{27R^2}{16r}\ or\ 16s^2r(R-r) \leq 27R^4$$

$$16\frac{27}{4}R^2.r(R-r) \overset{Mitrinovic}{\leq} 27R^4\ or\ R^2-4Rr+4r^2 \geq 0\ or\ (R-2r)^2 \geq 0\ true$$

$$We\ need\ to\ show:$$

$$\frac{s^2}{R^2}(R-r) \geq \frac{27r}{R}\ or, \frac{27Rr}{2R^2}(R-r) \overset{s^2 \geq \frac{27Rr}{2}}{\geq} \frac{27r}{R}\ or, 2(R-r) \geq R\ or, R \geq 2r\ true$$

Equality holds for an equilateral triangle.