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In ΔABC the following relationship holds:

$$\left(\sum_{cyc}\frac{ab}{a+b}\right)\left(3+\sum_{cyc}tan^2\left(\frac{A}{2}\right)\right)\le 3\sqrt{3}\frac{R^2}{r}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{ab}{a+b}\le\frac{1}{4}\sum_{cyc}\frac{(a+b)^2}{a+b}=\frac{1}{4}\sum_{cyc}(a+b)=p$$

$$3+\sum_{cyc}tan^2\left(\frac{A}{2}\right)=1+\frac{(4R+r)^2}{p^2}=\frac{p^2+(4R+r)^2}{p^2}$$

$$LHS\le p.\frac{p^2+(4R+r)^2}{p^2}\overset{Gerretsen}{\le}\frac{4R^2+4Rr+3r^2+16R^2+8Rr+r^2}{p}=$$

$$\frac{20R^2+12Rr+4r^2}{p}\overset{Euler}{\le}\frac{27R^2}{p}\overset{Mitrinovic}{\le}\frac{27R^2}{3\sqrt{3}r}=3\sqrt{3}\frac{R^2}{r}$$

$$Equality\ \ holds\ \ for\ \ a=b=c.$$