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In ΔABC the following relationship holds:

$$\frac{3r}{R} \le \sum_{cyc} \frac{a^2}{b^2+c^2} \le \frac{2R-r}{2r}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc} \frac{a^2}{b^2+c^2} \overbrace{\le}^{AM-GM} \sum_{cyc} \frac{a^2}{2bc} = \sum_{cyc} \frac{a^3}{2abc} =$$

$$= \frac{2(s^3-3sr^2-6sRr)}{2.4Rsr} = \frac{s^2-3r^2-6Rr}{4Rr} \overbrace{\le}^{Gerretsen}$$

$$\le \frac{4R^2+4Rr+3r^2-3r^2-6Rr}{4Rr} = \frac{4R^2-2Rr}{4Rr} = \frac{2R-r}{2r} \quad (*)$$

$$\sum_{cyc} \frac{a^2}{b^2+c^2} \overbrace{\ge}^{Nesbitt} \frac{3}{2} \overbrace{\ge}^{Euler} \frac{3r}{R} \quad (**)$$

$$From\ (*)\ and\ (**)\ we\ have:$$

$$\frac{3r}{R} \le \sum_{cyc} \frac{a^2}{b^2+c^2} \le \frac{2R-r}{2r}$$

$$Equality\ holds\ for: a=b=c$$