ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\left(3+\sum_{cyc}\frac{a^2+b^2}{c^2}\right)\left(\sum_{cyc} rr_a\right)\le\left(\frac{9}{4}\cdot\frac{R^2}{r}\right)^2$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$3+\sum_{cyc}\frac{a^2+b^2}{c^2}=\frac{a^2+b^2+c^2}{a^2}+\frac{a^2+b^2+c^2}{b^2}+\frac{a^2+b^2+c^2}{c^2}=$$

$$=(a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\overset{\substack{Leibniz\\ Steining}}{\le} 9R^2.\frac{1}{4r^2}=\frac{9R^2}{4r^2}\ (1)$$

$$\sum_{cyc} rr_a=r\sum_{cyc} r_a=r(4R+r)\overset{Euler}{\le}\frac{9R^2}{4}\ \ (2)$$

$$From\ (1)\ and\ (2)\ we\ have$$

$$\left(3+\sum_{cyc}\frac{a^2+b^2}{c^2}\right)\left(\sum_{cyc} rr_a\right)\overset{\substack{(1)\\(2)}}{\le}\left(\frac{9}{4}\cdot\frac{R^2}{r}\right)^2$$

$$Equality\ holds\ for: a=b=c$$