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In ΔABC the following relationship holds:

$$\cos\left(\frac{A-B}{2}\right).\cos\left(\frac{B-C}{2}\right).\cos\left(\frac{C-A}{2}\right)\le\frac{1}{4}\left(\frac{R}{r}+\frac{r}{R}\right)+\frac{3}{8}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$By\ \ Mollweide's\ \ formula:$$

$$\cos\left(\frac{A-B}{2}\right)=\frac{a+b}{c}.\sin\left(\frac{C}{2}\right)$$

$$\prod_{cyc}\cos\left(\frac{A-B}{2}\right)=\prod_{cyc}\frac{a+b}{c}.\prod_{cyc}\sin\left(\frac{C}{2}\right)=\frac{r}{4R}\left(2+\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)\right)\le$$

$$\le\left(2+3.\frac{R}{r}\right).\frac{r}{4R}=\frac{r}{2R}+\frac{3}{4}$$

$$Let's\ \ prove\ \ that\ :$$

$$\frac{r}{2R}+\frac{3}{4}\le\frac{1}{4}\left(\frac{R}{r}+\frac{r}{R}\right)+\frac{3}{8}$$

$$\frac{2r}{R}+3\le\frac{R}{r}+\frac{r}{R}+\frac{3}{2}\ \rightarrow\ 2R^2-3Rr-2r^2\ge 0\rightarrow$$

$$(R-2r)(2R+r)\ge 0\rightarrow\quad R\ge 2r\ \ (Euler)\ \ True$$

$$Equality\ \ holds\ \ for\ \ an\ \ equilateral\ \ triangle.$$