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In any Δ ABC the following relationship holds :

$$3+\sum_{cyc}\frac{b+c}{\sqrt{4r^2+(b-c)^2}}>\frac{1}{r}\sum_{cyc}h_a$$

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Solution by Soumava Chakraborty-Kolkata-India

$$3+\sum_{cyc}\frac{b+c}{\sqrt{4r^2+(b-c)^2}}\overset{\text{Bogdan Fustei}}{=}3+\sum_{cyc}\frac{s(b+c)}{an_a}$$

$$=3+\sum_{cyc}\frac{\sqrt{s^2-2h_ar_a+2h_ar_a}.(b+c)}{an_a}\overset{\text{Bogdan Fustei}}{=}3+\sum_{cyc}\frac{\sqrt{n_a^2+2h_ar_a}.(b+c)}{an_a}$$

$$>3+\sum_{cyc}\frac{n_a.(b+c)}{an_a}=\sum_{cyc}\left(1+\frac{b+c}{a}\right)=\frac{1}{r}\sum_{cyc}\frac{2rs}{a}=\frac{1}{r}\sum_{cyc}h_a$$

$$\therefore 3+\sum_{cyc}\frac{b+c}{\sqrt{4r^2+(b-c)^2}}>\frac{1}{r}\sum_{cyc}h_a\ \forall\ \Delta\ ABC\ (QED)$$