ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{\sin A+\sin B-\sin C}{\cos A+\cos B-\cos C+1}=\tan\frac{A}{2}\tan\frac{B}{2}\cot\frac{C}{2}$$

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Solution by Daniel Sitaru-Romania

$$\frac{\sin A+\sin B-\sin C}{\cos A+\cos B-\cos C+1}=\frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}-\sin\left(2\cdot\frac{C}{2}\right)}{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}-\cos\left(2\cdot\frac{C}{2}\right)+1}=$$

$$=\frac{2\sin\frac{\pi-C}{2}\cos\frac{A-B}{2}-2\sin\frac{C}{2}\cos\frac{C}{2}}{2\cos\frac{\pi-C}{2}\cos\frac{A-B}{2}+2\sin^2\frac{C}{2}}=\frac{2\cos\frac{C}{2}\cos\frac{A-B}{2}-2\sin\frac{C}{2}\cos\frac{C}{2}}{2\sin\frac{C}{2}\cos\frac{A-B}{2}+2\sin^2\frac{C}{2}}=$$

$$=\frac{\left(\cos\frac{A-B}{2}-\sin\frac{C}{2}\right)}{\left(\cos\frac{A-B}{2}+\sin\frac{C}{2}\right)}\cdot\frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}=\frac{\left(\cos\frac{A-B}{2}-\cos\frac{\pi-C}{2}\right)}{\left(\cos\frac{A-B}{2}+\cos\frac{\pi-C}{2}\right)}\cdot\cot\frac{C}{2}=$$

$$=\frac{2\sin\frac{\pi-C+A-B}{4}\sin\frac{\pi-C-A+B}{4}}{2\cos\frac{\pi-C+A-B}{4}\cos\frac{A-B-\pi+C}{4}}\cdot\cot\frac{C}{2}=\frac{2\sin\frac{A+B+C-C+A-B}{4}\sin\frac{A+B+C-C-A+B}{4}}{2\cos\frac{A+B+C-C+A-B}{4}\cos\frac{A-B-A-B-C+C}{4}}\cdot\cot\frac{C}{2}=$$

$$=\frac{\sin\frac{2A}{4}\sin\frac{2B}{4}}{\cos\frac{2A}{4}\cos\frac{2B}{4}}\cdot\cot\frac{C}{2}=\frac{\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}}\cdot\cot\frac{C}{2}=\tan\frac{A}{2}\tan\frac{B}{2}\cot\frac{C}{2}$$