ROMANIAN MATHEMATICAL MAGAZINE

$$I-\ incenter\ in\ \Delta ABC\ ,A(2,2),B(6,4),C(4,8),M(8,6).$$

$$Find\ MI.$$

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Solution by Tapas Das-India

$We\ know\ that\ in\ \Delta ABC\ with\ vertices\ A(x_1,y_1),B(x_2,y_2),C(x_3,y_3):$

$$Incenter-I=\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

$$now\ A(x_1,y_1)=A(2,2),B(x_2,y_2)=B(6,4),C(x_3,y_3)=C(4,8)$$

$$a=BC=\sqrt{(6-4)^2+(4-2)^2}=2\sqrt{5},$$
$$b=CA=\sqrt{(4-2)^2+(8-2)^2}=2\sqrt{10},$$
$$c=AB=\sqrt{(6-2)^2+(4-2)^2}=2\sqrt{5}$$

$$I-Incenter=$$
$$=\left(\frac{2\sqrt{5}\times 2+2\sqrt{10}\times 6+2\sqrt{5}\times 4}{2\sqrt{5}+2\sqrt{5}+2\sqrt{10}},\frac{2\sqrt{5}\times 2+2\sqrt{10}\times 4+2\sqrt{5}\times 8}{2\sqrt{5}+2\sqrt{5}+2\sqrt{10}}\right)=(3\sqrt{2},6-\sqrt{2})$$

$$MI=\sqrt{\left(8-3\sqrt{2}\right)^2+\left(6-6+\sqrt{2}\right)^2}=\sqrt{64+18-48\sqrt{2}+2}=2\sqrt{21-12\sqrt{2}}$$