ROMANIAN MATHEMATICAL MAGAZINE

If $z \in \mathbb{C}, n \in \mathbb{N}$ then:

$$\boldsymbol{\cos^n z = \frac{1}{(\cos^2 a + sh^2 b)^{\frac{n}{2}}} e^{-inarctg(tgathb)}}$$

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Solution by proposer

$$\cos^n z = \left(\frac{e^{iz} + e^{-iz}}{2}\right)^n = \left(\frac{e^{i(a+ib)} + e^{-i(a+ib)}}{2}\right)^n =$$

$$= \left(\frac{e^{ia}e^{-b} + e^{-ia}e^{b}}{2}\right)^n = \left[\frac{e^{-b}}{2}(\cos a + i\sin a) + \frac{e^{b}}{2}(\cos a - i\sin a)\right]^n =$$

$$= \left[\cos a\left(\frac{e^b + e^{-b}}{2}\right) - i\sin a\left(\frac{e^b - e^{-b}}{2}\right)\right]^n = (\cos a\, chb - i\sin a\, shb)^n$$

Remark:

$$\mathrm{A - iB} = \frac{1}{\sqrt{A^2 + B^2}}\left(\cos\left(arctg\frac{B}{A}\right) - i\sin(arctg\frac{B}{A})\right)$$

$$(\cos a\, chb - i\sin a\, shb)^n =$$

$$= \frac{1}{\left(\sqrt{\cos^2 a\, ch^2 b + \sin^2 a\, sh^2 b}\right)^n}(\cos(arctgtgathb) - -i\sin(arctg(tgathb)^{\mathrm{n}}$$

Remark:

$$\cos^2 a\, ch^2 b + \sin^2 a\, sh^2 b = \cos^2 a\, ch^2 b + (1 - \cos^2 a) sh^2 b =$$

$$= \cos^2 a(ch^2 b - sh^2 b) + sh^2 b = \cos^2 a + sh^2 b$$

$$\cos^n z = \frac{1}{(\cos^2 a + sh^2 b)^{\frac{n}{2}}}(\cos(arctgtgathb) - i\sin(arctg(tgathb)^{\mathrm{n}} =$$

$$= \frac{1}{(\cos^2 a + sh^2 b)^{\frac{n}{2}}} e^{-inarctg(tgathb)}$$