ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0\ then\ prove$$

$$\frac{a^2+b^2}{a^2+b^2+2c^2}+\frac{b^2+c^2}{b^2+c^2+2a^2}+\frac{c^2+a^2}{c^2+a^2+2b^2}\geq\frac{3}{2}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{a^2+b^2}{a^2+b^2+2c^2}+\frac{b^2+c^2}{b^2+c^2+2a^2}+\frac{c^2+a^2}{c^2+a^2+2b^2}\geq\frac{3}{2}$$

$$Let\ a^2+b^2=x>0,\quad b^2+c^2=y>0,\quad c^2+a^2=z>0.Then:$$

$$a^2=\frac{z+x-y}{2},\ b^2=\frac{x+y-z}{2},\ c^2=\frac{z+y-x}{2}$$

$$\frac{x}{x+2c^2}+\frac{y}{y+2a^2}+\frac{z}{z+2b^2}=$$

$$\frac{x}{x+z+y-x}+\frac{y}{y+z+x-y}+\frac{z}{z+x+y-z}=$$

$$\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}\overset{Nesbitt}{\geq}\frac{3}{2}$$

$$Equality\ holds\ for\ a=b=c.$$