

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\sum_{cyc} \frac{a^2 + b^2}{a^2 + b^2 + 2c^2} \geq \frac{3}{2}$$

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Denote:

$$x = a^2 + b^2 + c^2$$

$$\begin{aligned} \sum_{cyc} \frac{a^2 + b^2}{a^2 + b^2 + 2c^2} &= \sum_{cyc} \frac{x - c^2}{x + c^2} = \sum_{cyc} \frac{-x - c^2 + 2x}{x + c^2} = \\ &= -3 + 2x \sum_{cyc} \frac{1}{x + c^2} \stackrel{\text{BERGSTROM}}{\geq} -3 + 2x \cdot \frac{(1 + 1 + 1)^2}{3x + a^2 + b^2 + c^2} = \\ &= -3 + 2x \cdot \frac{9}{3x + x} = -3 + \frac{9}{2} = \frac{3}{2} \end{aligned}$$

Equality holds for $a = b = c$.