

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ and $a + b = 2ab$ then:

$$\frac{a}{b^2} + \frac{b}{a^2} \geq 2$$

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$$\frac{a}{b^2} + \frac{b}{a^2} \geq 2 \Leftrightarrow a^3 + b^3 \geq 2a^2b^2$$

$$(a + b)(a^2 - ab + b^2) \geq 2a^2b^2, \quad 2ab(a^2 - ab + b^2) \geq 2a^2b^2$$

$$a^2 - ab + b^2 \geq ab, \quad a^2 - 2ab + b^2 \geq 0$$

$$(a - b)^2 \geq 0$$

Equality holds for $a = b$.