ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, a+b+c\le 3$ then:

$$\frac{a+b}{c^2}+\frac{b+c}{a^2}+\frac{a+c}{b^2}\ge 6$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{a+b}{c^2}+\frac{b+c}{a^2}+\frac{a+c}{b^2}\overset{AM-GM}{\ge}\frac{2\sqrt{ab}}{c^2}+\frac{2\sqrt{bc}}{a^2}+\frac{2\sqrt{ac}}{b^2}=$$

$$2\left(\frac{\sqrt{ab}}{c^2}+\frac{\sqrt{bc}}{a^2}+\frac{\sqrt{ac}}{b^2}\right)\overset{AM-GM}{\ge}6\left(\frac{\sqrt{(abc)^2}}{a^2b^2c^2}\right)^{\frac{1}{3}}=\frac{6}{\sqrt[3]{abc}}\ge\frac{6}{1}=6$$

$$\therefore\left(\sqrt[3]{abc}\le\frac{a+b+c}{3}\le\frac{3}{3}=1\right)$$

$$Equality\ \ holds\ for\ a=b=c=1$$