ROMANIAN MATHEMATICAL MAGAZINE

**If $a, b > 0$ and $a^2 + b^2 = 2$ then prove that :**

$$\frac{a^2}{\sqrt{a+3}} + \frac{b^2}{\sqrt{b+3}} \le 1$$

*Proposed by Nguyen Hung Cuong-Vietnam*

*Solution by Soumava Chakraborty-Kolkata-India*

$$\frac{a^2}{\sqrt{a+3}} \overset{?}{\le} \frac{15a^2+1}{32} \Leftrightarrow (a+3)(15a^2+1)^2 \overset{?}{\ge} 1024a^4$$

$$\Leftrightarrow (a-1)^2(225a^3+101a^2+7a+3) \overset{?}{\ge} 0 \to \text{true} \because a > 0$$

$$\therefore \frac{a^2}{\sqrt{a+3}} \le \frac{15a^2+1}{32} \text{ and } \frac{b^2}{\sqrt{b+3}} \le \frac{15b^2+1}{32} \text{ and so,}$$

$$\frac{a^2}{\sqrt{a+3}} + \frac{b^2}{\sqrt{b+3}} \le \frac{15(a^2+b^2)+2}{32} \overset{a^2+b^2=2}{=} \frac{32}{32} = 1$$

$$\therefore \frac{a^2}{\sqrt{a+3}} + \frac{b^2}{\sqrt{b+3}} \le 1 \ \forall \ a, b > 0 \ \Big| \ a^2 + b^2 = 2, '' = '' \text{ iff } a = b = 1 \text{ (QED)}$$