ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\frac{b-a}{\sqrt{b+1}}+\frac{c-b}{\sqrt{c+1}}+\frac{a-c}{\sqrt{a+1}} \leq 0$$

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Solution by Tapas Das-India

$$Let\ a+1=x^2, b+1=y^2, c+1=z^2$$

$$then\ b-a=y^2-x^2, c-b=z^2-y^2, a-c=x^2-z^2$$

$$\frac{b-a}{\sqrt{b+1}}+\frac{c-b}{\sqrt{c+1}}+\frac{a-c}{\sqrt{a+1}}=\frac{y^2-x^2}{y}+\frac{z^2-y^2}{z}+\frac{x^2-z^2}{x}=$$
$$=(x+y+z)-\left(\frac{x^2}{y}+\frac{y^2}{z}+\frac{z^2}{x}\right)\overset{Bergstrom}{\leq}(x+y+z)-\frac{(x+y+z)^2}{x+y+z}=0$$

Equality holds for a=b=c.