ROMANIAN MATHEMATICAL MAGAZINE

If $a,b>0, ab=1$ then:

$$\left(\frac{a}{b+1}\right)^2+\left(\frac{b}{a+1}\right)^2\ge\frac{1}{2}$$

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Solution by Tapas Das-India

$$\frac{a}{b+1}+\frac{b}{a+1}=\frac{a(a+1)+b(b+1)}{(a+1)(b+1)}=\frac{a^2+b^2+a+b}{1+ab+a+b}\overset{\substack{AM-GM\\ab=1}}{\ge}$$

$$\ge\frac{2ab+a+b}{ab+ab+a+b}=1\ (1)$$

$$\left(\frac{a}{b+1}\right)^2+\left(\frac{b}{a+1}\right)^2=\frac{\left(\frac{a}{b+1}\right)^2}{1}+\frac{\left(\frac{b}{a+1}\right)^2}{1}\overset{Radon}{\ge}\frac{\left(\frac{a}{b+1}+\frac{b}{a+1}\right)^2}{2}\overset{(1)}{\ge}\frac{1}{2}$$

Equality holds for a=b=1