ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{a}{\sqrt{2a^2+b^2+c^2}}+\frac{b}{\sqrt{2b^2+a^2+c^2}}+\frac{c}{\sqrt{2c^2+b^2+a^2}}\le\frac{3}{2}$$

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Solution by Tapas Das-India

$$2a^2+b^2+c^2=\left(\frac{a^2}{1}+\frac{b^2}{1}\right)+\left(\frac{a^2}{1}+\frac{c^2}{1}\right)\overset{Radon}{\ge}$$

$$\ge\frac{(a+b)^2}{2}+\frac{(a+c)^2}{2}\overset{AM-GM}{\ge}(a+b)(a+c)$$

$$\frac{a}{\sqrt{2a^2+b^2+c^2}}+\frac{b}{\sqrt{2b^2+a^2+c^2}}+\frac{c}{\sqrt{2c^2+b^2+a^2}}=$$

$$=\sum\frac{a}{\sqrt{2a^2+b^2+c^2}}\le\sum\frac{a}{\sqrt{(a+b)(a+c)}}=\sum\sqrt{\frac{a}{a+b}\cdot\frac{a}{a+c}}\overset{AM-GM}{\le}$$

$$\le\frac{1}{2}\sum\left(\frac{a}{a+b}+\frac{a}{a+c}\right)=\frac{1}{2}\sum\left(\frac{a}{a+b}+\frac{b}{a+b}\right)=\frac{1}{2}\sum\frac{a+b}{a+b}=\frac{3}{2}$$

Equality holds for an equilateral triangle.