

# ROMANIAN MATHEMATICAL MAGAZINE

If  $a, b > 0, a + b = 2$  then:

$$a(a^2 + b) + b(b^2 + a) \geq 4$$

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$$\begin{cases} a + b = S \\ ab = P \end{cases} \Rightarrow S = 2$$

$$2 = S = a + b \stackrel{AM-GM}{\geq} 2\sqrt{ab} = 2\sqrt{P} \Rightarrow P \leq 1 \quad (1)$$

$$\begin{aligned} a(a^2 + b) + b(b^2 + a) &\geq 4 \\ a^3 + b^3 + 2ab &\geq 4, \quad S^3 - 3SP + 2P \geq 4 \\ 8 - 6P + 2P &\geq 4, \quad 4P \leq 4 \stackrel{(1)}{\Leftrightarrow} P \leq 1 \end{aligned}$$

Equality holds for  $a = b$ .