If $a, b > 0, a + b = 2$ then:

$$a(a^2 + b) + b(b^2 + a) \geq 4$$

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$$\begin{cases} a + b = S \\ ab = P \end{cases} \Rightarrow S = 2$$

$$2 = S = a + b \overset{AM-GM}{\geq} 2\sqrt{ab} = 2\sqrt{P} \Rightarrow P \leq 1 \ (1)$$

$$a(a^2 + b) + b(b^2 + a) \geq 4$$
$$a^3 + b^3 + 2ab \geq 4, \ \ S^3 - 3SP + 2P \geq 4$$
$$8 - 6P + 2P \geq 4, \qquad 4P \leq 4 \overset{(1)}{\Longleftrightarrow} P \leq 1$$

Equality holds for $a = b$.