If $a, b, c > 0, abc = 1$ then:

$$a + b + c + \frac{1}{ab + bc + ca} \geq \frac{10}{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$abc = 1 \text{ then } a + b + c \stackrel{AM-GM}{\geq} 3\sqrt[3]{abc} = 3$$

$$We \ need \ to \ show:$$
$$a + b + c + \frac{1}{ab + bc + ca} \geq \frac{10}{3}$$

$$a + b + c + \frac{1}{\frac{(a+b+c)^2}{3}} \geq \frac{10}{3}$$

$$x + \frac{3}{x^2} \stackrel{x=a+b+c \geq 3}{\geq} \frac{10}{3}$$
$$3x^3 - 10x^2 + 9 \geq 0$$

$$(x - 3)(3x^2 - x - 3) \geq 0 \ true \ as \ x \geq 3$$

$$Equality \ holds \ for \ a = b = c = 1.$$