ROMANIAN MATHEMATICAL MAGAZINE

If $a,b>0$ then:

$$\frac{a^6+b^6}{a^3\,b^3}+\frac{2ab}{a^2+b^2}\geq 3$$

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$$Let\ x=\frac{a^2+b^2}{ab}\overset{AM-GM}{\geq}\frac{2ab}{ab}=2\ (1)$$

$$\frac{a^6+b^6}{a^3\,b^3}=\frac{(a^2+b^2)^3-3a^2b^2(a^2+b^2)}{a^3b^3}=\left(\frac{a^2+b^2}{ab}\right)^3-3\frac{a^2+b^2}{ab}=x^3-3x$$

$$We\ need\ to\ show:$$

$$\frac{a^6+b^6}{a^3\,b^3}+\frac{2ab}{a^2+b^2}\geq 3$$

$$or,x^3-3x+\frac{2}{x}\geq 3\ or,x^4-3x^2-3x+2\geq 0\ or,(x-2)(x^3+2x^2+x-1)\geq 0$$

$$true\ as\ x\geq 2$$

Equality holds for a=b=1.