ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 1$ and $a + b + c = 6$ then prove that :

$$\frac{a-1}{b^2}+\frac{b-1}{c^2}+\frac{c-1}{a^2}\geq\frac{3}{4}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\frac{a-1}{b^2}=\sum_{cyc}\frac{(a-1)^3}{(ab-b)^2}\overset{Radon}{\geq}\frac{\left(\sum_{cyc}a-3\right)^3}{\left(\sum_{cyc}ab-\sum_{cyc}a\right)^2}\geq\frac{\left(\sum_{cyc}a-3\right)^3}{\left(\frac{\left(\sum_{cyc}a\right)^2}{3}-\sum_{cyc}a\right)^2}$$

$$\overset{a+b+c=6}{=}\frac{3^3}{(12-3)^2}=\frac{3}{4}\therefore\frac{a-1}{b^2}+\frac{b-1}{c^2}+\frac{c-1}{a^2}\geq\frac{3}{4}\ \forall\, a,b,c>1\,\Big|\,a+b+c=6,$$

$$''=''\ \text{iff } a=b=c=2\ (\text{QED})$$