ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0,\ a+b+c=3$ then:

$$\frac{a^2-1}{b+c}+\frac{b^2-1}{a+c}+\frac{c^2-1}{a+b}\geq 0$$

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$$a, b, c > 0$$

$$a+b+c=3 \to \begin{cases} a+b=3-c \\ a+c=3-b \\ b+c=3-a \end{cases}$$

$$\frac{a^2-1}{3-a}+\frac{b^2-1}{3-b}+\frac{c^2-1}{3-c}\geq 0 \to tangent\ line\ method\ f(x)=\frac{x^2-1}{3-x}$$

$$(x-1)^2\geq 0\ \ x^2-2x+1\geq 0\ \ 2x^2-4x+2\geq 0$$

$$x^2-1+x^2-4x+3\geq 0 \to\ x^2-1+(x-1)(x-3)\geq 0$$

$$x^2-1\geq (x-1)(3-x) \to \frac{x^2-1}{3-x}\geq x-1$$

$$\frac{a^2-1}{3-a}\geq a-1 \to \sum_{cyc}\frac{a^2-1}{b+c}\geq (a-1)+(b-1)+(c-1)$$

$$\sum_{cyc}\frac{a^2-1}{b+c}\geq a+b+c-3=0$$

$$Equality\ holds\ for\ a=b=c=1.$$