ROMANIAN MATHEMATICAL MAGAZINE

If $4x^3y^3z^3 \geq x^2+y^2+z^2+1$ then:

$$x^2y^2z^2 \geq 3$$

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Solution by Amin Hajiyev-Azerbaijan

$$\frac{x^2+y^2+z^2+1}{4} \overset{AM-GM}{\geq} \sqrt[4]{x^2y^2z^2}$$

$$x^2+y^2+z^2+1 \geq 4\sqrt{xyz}$$

$$4x^3y^3z^3 \geq 4\sqrt{xyz} \to x^5y^5z^5 \geq 1$$

$$xyz \geq 1$$

$$\frac{x^2+y^2+z^2}{3} \overset{AM-GM}{\geq} \sqrt[3]{x^2y^2z^2} \to x^2+y^2+z^2 \geq 3$$

Equality holds if and only if

$$x=y=z=1.$$