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If $a,b,c>0$ then:

$$\sum\frac{a}{b+c}-\frac{4abc}{2abc+\sum ab(a+b)}\geq 1$$

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Solution by Tapas Das-India

$$2abc+\sum ab(a+b)=2abc+(a+b+c)(ab+bc+ca)-3abc=$$

$$=(a+b+c)(ab+bc+ca)-abc=$$

$$=(a+b)(b+c)(c+a)\overset{Cesaro}{\geq}8abc\ (1)$$

$$\sum\frac{a}{b+c}-\frac{4abc}{2abc+\sum ab(a+b)}\overset{Nesbitt\ \&\ (1)}{\geq}\frac{3}{2}-\frac{4abc}{8abc}=1$$

Equality holds for a=b=c.