ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\sum \frac{a^{12}}{a^{12} + (b^3 + c^3)a^3b^3c^3} \geq 1$$

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We know that:

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2) \overset{AM-GM}{\geq} (x + y)(2xy - xy) = xy(x + y)$$

$$b^3c^3(b^3 + c^3) \leq (b^3)^3 + (c^3)^3 = b^9 + c^9$$

$$a^{12} + a^3\, b^3c^3(b^3 + c^3) \leq a^{12} + a^3(b^9 + c^9) = a^3(a^9 + b^9 + c^9)$$

$$\sum \frac{a^{12}}{a^{12} + (b^3 + c^3)a^3b^3c^3} \geq \sum \frac{a^{12}}{a^3(a^9 + b^9 + c^9)} = \sum \frac{a^9}{(a^9 + b^9 + c^9)} = 1$$

Equality holds for a=b=c.