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If $a, b, c > 0$ then:

$$4\sum\frac{a^8+b^8}{(a^2+b^2)^2}+abc\sum a\geq\sum a\sum a^3$$

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We know that by Schur's inequality if $a, b, c > 0$ then:

$$a^t(a-b)(a-c)+b^t(b-c)(b-a)+c^t(c-a)(c-b)\geq 0$$

For $t=2$ we have $a^2(a-b)(a-c)+b^2(b-c)(b-a)+c^2(c-a)(c-b)\geq 0$

$$\sum\left(a^4-a^3(b+c)+a^2bc\right)\geq 0 \text{ or } \sum a^4+abc\sum a\geq\sum a^3(b+c)\ \ (1)$$

$$a^4+b^4=\frac{(a^2)^2}{1}+\frac{(b^2)^2}{1}\overset{Radon}{\geq}\frac{(a^2+b^2)^2}{2} \text{ or } 2(a^4+b^4)\geq(a^2+b^2)^2\ (2)$$

$$a^8+b^8=\frac{(a^4)^2}{1}+\frac{(b^4)^2}{1}\overset{Radon}{\geq}\frac{(a^4+b^4)^2}{2}\ (3)$$

$$4\sum\frac{a^8+b^8}{(a^2+b^2)^2}+abc\sum a\overset{(3)\&(2)}{\geq}\sum(a^4+b^4)+abc\sum a=2\sum a^4+abc\sum a$$

$$\sum a^4+\left(\sum a^4+abc\sum a\right)\overset{(1)}{\geq}\sum a^4+\sum a^3(b+c)=$$

$$=\sum a^4+\sum a^3(a+b+c-a)=\sum a^4+\sum a^3\sum a-\sum a^4=\sum a\sum a^3$$

Equality holds for a=b=c.