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If $a,b,c>0, n\in\mathbb{N}, \sum_{cyc} a^2 \ge 3$ then:

$$\sum_{cyc}\left(\frac{a^4}{a^2+2b+2c}\right)^n \ge \frac{3}{5^n}$$

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$$a^2+1\ge 2a \Rightarrow \sum_{cyc} 2a \le \sum_{cyc}(a^2+1) \Rightarrow 2\sum_{cyc} a \le \sum_{cyc} a^2 + 3$$

$$\sum_{cyc}\frac{a^4}{a^2+2b+2c} \underbrace{\ge}^{\text{BERGSTROM}} \frac{(\sum a^2)^2}{\sum(a^2+2b+2c)} = \frac{(\sum a^2)^2}{\sum a^2 + 4\sum a} \ge$$

$$\ge \frac{(\sum a^2)^2}{\sum a^2 + 2\sum a^2 + 6} = \frac{(\sum a^2)^2}{3\sum a^2 + 6} \underbrace{\ge}^{6\le 2\sum a^2} \frac{(\sum a^2)^2}{3\sum a^2 + 2\sum a^2} = \frac{\sum a^2}{5} \underbrace{\ge}^{\sum a^2 \ge 3} = \frac{3}{5}$$

$$\frac{1}{3}\cdot\sum_{cyc}\left(\frac{a^4}{a^2+2b+2c}\right)^n \underbrace{\ge}^{\text{Power Mean}} \left(\frac{1}{3}\cdot\sum_{cyc}\frac{a^4}{a^2+2b+2c}\right)^n \ge \left(\frac{1}{3}\cdot\frac{3}{5}\right)^n = \frac{1}{5^n}$$

$$\sum_{cyc}\left(\frac{a^4}{a^2+2b+2c}\right)^n \ge \frac{3}{5^n}$$

Equality holds if and only if $a=b=c=1$.