

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$, $\sum_{\text{cyc}} \frac{1}{x} = 3$ then :

$$\sum_{\text{cyc}} \frac{1}{x^3 + x + 2} \leq \frac{3}{4}$$

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$$\frac{1}{x^3 + x + 2} \stackrel{?}{\leq} \frac{1}{4x} \Leftrightarrow x^3 - 3x + 2 \stackrel{?}{\geq} 0 \Leftrightarrow (x - 1)^2(x + 2) \stackrel{?}{\geq} 0$$

$$\begin{aligned} \rightarrow \text{true } \because x > 0 \therefore \frac{1}{x^3 + x + 2} &\leq \frac{1}{4x} \text{ and analogs } \therefore \sum_{\text{cyc}} \frac{1}{x^3 + x + 2} \leq \frac{1}{4} \left(\sum_{\text{cyc}} \frac{1}{x} \right) \\ &= \frac{3}{4} \forall x, y, z > 0 \mid \sum_{\text{cyc}} \frac{1}{x} = 3, " = " \text{ iff } x = y = z = 1 \text{ (QED)} \end{aligned}$$