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If $a, \mathrm{b}, \mathrm{c} > 0, a + \mathrm{b} + \mathrm{c} = 1$ *and* $\lambda \geq \frac{6}{5}$ then :

$$\sum_{cyc} a^2 \leq \frac{3-\lambda}{9} + \lambda \sum_{cyc} a^3$$

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$$\sum_{cyc} a^2 \overset{?}{\leq} \frac{3-\lambda}{9} + \lambda \sum_{cyc} a^3 \overset{a+b+c=1}{\Leftrightarrow} \left(\sum_{cyc} a^2\right)\left(\sum_{cyc} a\right) \overset{?}{\leq}$$

$$\frac{3-\lambda}{9}.\left(\sum_{cyc} a\right)^3 + \lambda \sum_{cyc} a^3$$

$$\Leftrightarrow \left(\sum_{cyc} a^2\right)\left(\sum_{cyc} a\right) \overset{?}{\leq} \frac{1}{3}.\left(\sum_{cyc} a\right)^3 + \frac{\lambda}{9}\left(9\sum_{cyc} a^3 - \left(\sum_{cyc} a\right)^3\right) \text{ and } \because$$

$$9\sum_{cyc} a^3 - \left(\sum_{cyc} a\right)^3 \overset{\text{Holder}}{\geq} 0 \text{ and } \because \lambda \geq \frac{6}{5} \therefore \text{ it suffices to prove :}$$

$$\left(\sum_{cyc} a^2\right)\left(\sum_{cyc} a\right) \overset{?}{\leq} \frac{1}{3}.\left(\sum_{cyc} a\right)^3 + \frac{\left(\frac{6}{5}\right)}{9}\left(9\sum_{cyc} a^3 - \left(\sum_{cyc} a\right)^3\right)$$

$$\Leftrightarrow 6\sum_{cyc} a^3 + \left(\sum_{cyc} a\right)^3 - 5\left(\sum_{cyc} a^2\right)\left(\sum_{cyc} a\right) \overset{?}{\geq} 0$$

$$\Leftrightarrow \sum_{cyc} a^3 + 3abc \overset{?}{\geq} \sum_{cyc} a^2 b + \sum_{cyc} ab^2 \rightarrow \text{true via Schur} \therefore \sum_{cyc} a^2 \leq \frac{3-\lambda}{9} + \lambda \sum_{cyc} a^3$$

$$\forall\, a, b, c > 0 \,\middle|\, a + b + c = 1 \text{ and } \lambda \geq \frac{6}{5}, ''='' \text{ iff } a = b = c = \frac{1}{3}.$$