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If $a, b, c > 0, a^2 + b^2 + c^2 = 3$ *and* $\lambda \geq 0$ then :

$$\sum_{cyc}\left(a.\sqrt{\lambda + b + c}\right) \leq \sqrt{\lambda + 2}(a + b + c)$$

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$$\sum_{cyc}\left(a.\sqrt{\lambda + b + c}\right) = \sum_{cyc}\left(\sqrt{a}.\sqrt{\lambda a + ab + ca}\right) \overset{CBS}{\leq}$$

$$\leq \sqrt{\sum_{cyc} a}.\sqrt{\lambda\sum_{cyc} a + 2\sum_{cyc} ab} \overset{?}{\leq} \sqrt{\lambda + 2}.\left(\sum_{cyc} a\right)$$

$$\Leftrightarrow \lambda\left(\sum_{cyc} a\right)^2 + 2\left(\sum_{cyc} a\right)\left(\sum_{cyc} ab\right) \overset{?}{\leq} \lambda\left(\sum_{cyc} a\right)^2 + 2\left(\sum_{cyc} a\right)^2 \Leftrightarrow \sum_{cyc} a \overset{?}{\geq} \sum_{cyc} ab$$

$$\Leftrightarrow \left(\sum_{cyc} a\right).\sqrt{\frac{1}{3}\sum_{cyc} a^2} \overset{?}{\geq} \sum_{cyc} ab \left(\because \sum_{cyc} a^2 = 3\right)$$

$$\Leftrightarrow \left(\sum_{cyc} a^2\right)\left(\sum_{cyc} a\right)^2 \overset{?}{\geq} 3\left(\sum_{cyc} ab\right)^2 \rightarrow \text{true} \because \sum_{cyc} a^2 \geq \sum_{cyc} ab \text{ and}$$

$$\left(\sum_{cyc} a\right)^2 \geq 3\sum_{cyc} ab \therefore \sum_{cyc}\left(a.\sqrt{\lambda + b + c}\right) \leq \sqrt{\lambda + 2}.(a + b + c)$$

$$\forall\, a, b, c > 0 \,\middle|\, a^2 + b^2 + c^2 = 3 \text{ and } \lambda \geq 0, '' ='' \text{ iff } a = b = c = 1 \text{ (QED)}$$