ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,|\lambda|\leq 2$ then:

$$\sum(b+c)(a^2+\lambda ab+b^2)\geq\frac{3(\lambda+2)}{4}\prod(a+b)$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$a^2+\lambda ab+b^2=(a+b)^2-2ab+\lambda ab=(a+b)^2+(\lambda-2)ab\overset{\substack{AM-GM\\ as\ |\lambda|\leq 2}}{\geq}$$

$$\geq(a+b)^2+(\lambda-2)\frac{(a+b)^2}{4}=\frac{(\lambda+2)}{4}(a+b)^2$$

$$\sum(b+c)(a^2+\lambda ab+b^2)\geq\sum(b+c)\frac{(\lambda+2)}{4}(a+b)^2\overset{AM-GM}{\geq}$$

$$\geq\frac{3(\lambda+2)}{4}\sqrt[3]{\prod(b+c)(a+b)^2}=\frac{3(\lambda+2)}{4}\prod(a+b)$$

Equality holds for a=b=c.