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If $a,b,c>0$, $a+b+c=3$, $n\in N$ then:

$$\sum\left(\frac{a^3}{a^2+b^2}\right)^n\ge\frac{3}{2^n}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum\frac{a^3}{a^2+b^2}=\sum\left(a-\frac{ab^2}{a^2+b^2}\right)=\sum a-\sum\frac{ab^2}{a^2+b^2}\overset{AM-GM}{\ge}$$

$$\ge\sum a-\sum\frac{ab^2}{2ab}=\sum a-\frac{1}{2}\sum a\overset{a+b+c=3}{=}\frac{3}{2}\quad(1)$$

$$\sum\left(\frac{a^3}{a^2+b^2}\right)^n=\sum\frac{\left(\frac{a^3}{a^2+b^2}\right)^n}{(1)^{n-1}}\overset{Radon}{\ge}\frac{\left(\sum\frac{a^3}{a^2+b^2}\right)^n}{3^{n-1}}\overset{(1)}{\ge}\frac{\left(\frac{3}{2}\right)^n}{3^{n-1}}=\frac{3}{2^n}$$

Equality holds for a=b=c=1