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If $a, \mathrm{b}, \mathrm{c} > 0, a + \mathrm{b} + \mathrm{c} = 3$ and $\lambda \geq 1, \mathrm{n} \geq 0$ then :

$$\sum_{cyc} \frac{a^2 - a + \lambda}{\mathrm{b} + \mathrm{n}} \geq \frac{3\lambda}{\mathrm{n} + 1}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} \frac{a^2 - a + \lambda}{\mathrm{b} + \mathrm{n}} = \sum_{cyc} \frac{a^2 - a + 1}{\mathrm{b} + \mathrm{n}} + \sum_{cyc} \frac{\lambda - 1}{\mathrm{b} + \mathrm{n}} \geq$$

$$\sum_{cyc} \frac{\frac{1}{4}(a+1)^2}{\mathrm{b} + \mathrm{n}} + (\lambda - 1). \sum_{cyc} \frac{1}{\mathrm{b} + \mathrm{n}} \overset{\text{Bergstrom}}{\geq} \frac{\frac{1}{4}\left(\sum_{cyc} a + 3\right)^2}{\sum_{cyc} a + 3\mathrm{n}} + (\lambda - 1). \frac{9}{\sum_{cyc} a + 3\mathrm{n}}$$

$$(\because a, \mathrm{b}, \mathrm{c} > 0 \wedge \mathrm{n} \geq 0 \Rightarrow \mathrm{b} + \mathrm{n}, \mathrm{c} + \mathrm{n}, a + \mathrm{n} > 0 \text{ and } \because \lambda - 1 \geq 0)$$

$$= \frac{9}{3 + 3\mathrm{n}} + \frac{9(\lambda - 1)}{3 + 3\mathrm{n}} = \frac{3\lambda}{\mathrm{n} + 1} \therefore \sum_{cyc} \frac{a^2 - a + \lambda}{\mathrm{b} + \mathrm{n}} \geq \frac{3\lambda}{\mathrm{n} + 1}$$

$$\forall a, \mathrm{b}, \mathrm{c} > 0 \,\middle|\, a + \mathrm{b} + \mathrm{c} = 3 \text{ and } \lambda \geq 1, \mathrm{n} \geq 0, '' ='' \text{ iff } a = \mathrm{b} = \mathrm{c} = 1 \text{ (QED)}$$