ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0\ and\ a^2+b^2+c^2=3\ then:$$

$$\frac{1}{1+ab}+\frac{1}{1+bc}+\frac{1}{1+ac}\ge\frac{3}{2}$$

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Cauchy-Schwarz inequality:

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2\le\left(\sum_{i=1}^{n}a_i^2\right)\left(\sum_{i=1}^{n}b_i^2\right)\ as\ a_i,b_i\in R$$

$$a_1=\frac{1}{b_1}=\sqrt{1+ab},a_2=\frac{1}{b_2}=\sqrt{1+bc},a_3=\frac{1}{b_3}=\sqrt{1+ac}$$

$$(a_1\cdot b_1+a_2\cdot b_2+a_3\cdot b_3)^2\le\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)$$

$$3^2\le(3+ab+bc+ac)\left(\frac{1}{1+ab}+\frac{1}{1+bc}+\frac{1}{1+ac}\right)$$

$$\sum_{cyc}\frac{1}{1+ab}\ge\frac{9}{3+ab+ac+bc}$$

$$We\ know\ (a-b)^2+(b-c)^2+(a-c)^2\ge 0$$

$$a^2+b^2+c^2\ge ab+bc+ac\to 3\ge ab+ac+bc$$

$$\sum_{cyc}\frac{1}{1+ab}\ge\frac{9}{6}=\frac{3}{2}$$

$$\frac{1}{1+ab}+\frac{1}{1+bc}+\frac{1}{1+ac}\ge\frac{3}{2}$$

$$Equality\ holds\ for\ a=b=c=1$$