If $x,y,z>0$ then:

$$\frac{x+2\sqrt{yz}}{y+z}+\frac{y+2\sqrt{zx}}{z+x}+\frac{z+2\sqrt{xy}}{x+y}\geq 3$$

Proposed by Gheorghe Crăciun-Romania

Solution 1 by Kasem Abotrabi, Khaled Abd Imouti-Syria

$$L=\frac{x+2\sqrt{yz}}{y+z}+\frac{y+2\sqrt{zx}}{z+x}+\frac{z+2\sqrt{xy}}{x+y}\geq 3$$

Suppose : $x=a^2, y=b^2, z=c^2 \Rightarrow \sqrt{xy}=ab, \sqrt{zx}=ac, \sqrt{yz}=bc$

$$SO:\ L=\frac{c^2+2ab}{a^2+b^2}+\frac{b^2+2ac}{a^2+c^2}+\frac{a^2+2cb}{c^2+b^2}\geq 3$$

$$\frac{c^2+2ab}{a^2+b^2}=\frac{c^2+2ab-(a^2+b^2)+(a^2-b^2)}{a^2+b^2}=1+\frac{c^2-(a-b)^2}{a^2+b^2}$$

So let : $L=\frac{c^2-(a-b)^2}{a^2+b^2}+\frac{b^2-(c-a)^2}{c^2+a^2}+\frac{a^2-(b-c)^2}{b^2+c^2}\geq 0$

$$\frac{c^2}{a^2+b^2}=\frac{a^2+b^2+c^2-(a^2+b^2)}{a^2+b^2}=\frac{a^2+b^2+c^2}{a^2+b^2}-1$$

$$L=(a^2+b^2+c^2)\left(\frac{1}{a^2+b^2}+\frac{1}{c^2+a^2}+\frac{1}{b^2+c^2}\right)-3+\left(\frac{1}{a^2+b^2}+\frac{1}{c^2+a^2}+\frac{1}{b^2+c^2}\right)\geq 6$$

$$L=1+\frac{a^2+b^2+c^2+2ab}{a^2+b^2}+1+\frac{a^2+b^2+c^2+2ca}{a^2+c^2}+1+\frac{a^2+b^2+c^2+2cb}{b^2+c^2}\geq?6+3$$

$$L=\frac{a^2+b^2+c^2+(a+b)^2}{a^2+b^2}+\frac{a^2+b^2+c^2+(a+c)^2}{a^2+c^2}+\frac{a^2+b^2+c^2+(b+c)^2}{b^2+c^2}\geq?9$$

$$L\geq\frac{(a+b+c+a+b+a+b+c+a+c+a+b+c+b+c)^2}{2a^2+2b^2+2c^2}$$

$$L\geq\frac{25(a+b+c)^2}{2(a^2+b^2+c^2)}\geq\frac{25}{2}.\frac{3}{2}=\frac{75}{4}\geq?9\ \ True$$

(Bergstrom's inequality)

Equality holds when a = b = c =1

Solution 2 by Kasem Abotrabi, Khaled Abd Imouti-Syria

$$x,y,z>0$$

$$A=\frac{x+2\sqrt{yz}}{y+z}+\frac{y+2\sqrt{zx}}{z+x}+\frac{z+2\sqrt{xy}}{x+y}\geq?3$$

ROMANIAN MATHEMATICAL MAGAZINE

$$A=\sum_{cyc}\frac{x}{y+z}+\sum_{cyc}\frac{2\sqrt{yz}}{y+z}$$

$$A=\sum_{cyc}\frac{(\sqrt{x})^2}{y+z}+\sum_{cyc}\frac{2\sqrt{yz}+y+z}{y+z}-3$$

$$A=\sum_{cyc}\frac{(\sqrt{x})^2}{y+z}+\sum_{cyc}\frac{(\sqrt{y}+\sqrt{z})^2}{y+z}-3 \qquad (\text{Bergstrom's inequality})$$

$$A\ge\frac{(\sqrt{x}+\sqrt{y}+\sqrt{z})^2}{x+y+z}+\frac{(2(\sqrt{x}+\sqrt{y}+\sqrt{z}))^2}{x+y+z}-3$$

$$A\ge\frac{5\left(\sqrt{x}+\sqrt{y}+\sqrt{z}\right)^2}{2(x+y+z)}-3 \qquad .\ AM-GM$$

$$\sqrt{x}+\sqrt{y}+\sqrt{z}\ge 3\sqrt[3]{\sqrt{xyz}} \quad , x+y+z\ge 3\sqrt[3]{xyz}$$

$$A\ge\frac{5\left(3\sqrt[3]{\sqrt{xyz}}\right)^2}{2\left(3\sqrt{xyz}\right)}-3\Rightarrow A\ge\frac{45\sqrt[3]{xyz}}{6\sqrt[3]{xyz}}-3\Rightarrow$$

$$A\ge\frac{15}{2}-3\Rightarrow A\ge\frac{9}{2}$$

Equality holds when x = y = z =1