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If $a,b,c,d>0,\ a+b+c+d=4$ then:

$$\sum_{cyc}\sqrt{\frac{a}{4-a}}>\frac{8abcd}{ab(c+d)+ca(a+b)}$$

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$$\text{Showing that when}:\ 0<a<4,\quad \sqrt{\frac{a}{4-a}}\ge\frac{a}{2}$$

$$\text{So that}:\ \frac{a}{4-a}\ge\frac{a^2}{4}\to\ a^2-4a+4\ge0\ \to\ (a-2)^2\ge0$$

$$\sum_{cyc}\sqrt{\frac{a}{4-a}}\ge\frac{1}{2}(a+b+c+d)$$

$$\text{Let us prove that}:$$

$$\frac{1}{2}(a+b+c+d)>\frac{8abcd}{ab(c+d)+ca(a+b)}$$

$$or:\ (a+b+c+d)\big(ab(c+d)+ca(a+b)\big)>16abcd$$

$$LHS=(a+b+c+d)\big(ab(c+d)+ca(a+b)\big)\overset{AM-GM}{\ge}$$

$$\ge4\sqrt[4]{abcd}\big(2ab\sqrt{cd}+2cd\sqrt{ab}\big)\overset{AM-GM}{\ge}4\sqrt[4]{abcd}.2(2\sqrt{(abcd)(abcd)}=$$

$$=16\sqrt[4]{abcd}.\sqrt{(abcd)}.\sqrt[4]{abcd}=16abcd$$

$$\text{According to the } a,b,c,d>0 \text{ and } a+b+c+d=4$$

$$\text{the equality condition is not satisfied.}$$