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If $a,b,c>0$ then:

$$\frac{a}{\sqrt{b+c}}+\frac{b}{\sqrt{a+c}}+\frac{c}{\sqrt{a+b}}>\sqrt{\frac{a+b+c}{2}}$$

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Lemma 1. *Holder's inequality.*

$$\left(\sum_{i=1}^{n}\frac{x_i}{\sqrt{y_i}}\right)^2\left(\sum_{i=1}^{n}x_iy_i\right)\geq\left(\sum_{i=1}^{n}x_i\right)^3,\qquad x_i,y_i>0$$

Lemma 2. $a^2+b^2+c^2\geq ab+ac+bc$

$$a^2+b^2+c^2-(ab+ac+bc)=\frac{1}{2}\left(\sum_{cyc}(a-b)^2\right)\geq 0$$

$$(a+b+c)^2=\underbrace{a^2+b^2+c^2}_{\geq ab+ac+bc}+2(ab+ac+bc)$$

$$(a+b+c)^2\geq 3(ab+ac+bc)$$

$$\begin{cases}x_1=a,x_2=b,x_3=c\\ y_1=c+b,y_2=a+c,y_3=b+a\end{cases}$$

$$\left(\sum_{cyc}\frac{a}{\sqrt{b+c}}\right)^2\left(\sum_{cyc}a(b+c)\right)\geq(a+b+c)^3$$

$$LHS^2\geq\frac{(a+b+c)^3}{\sum_{cyc}a(b+c)}=\frac{(a+b+c)^3}{2(ab+ac+bc)}$$

$$ab+ac+bc\underset{\text{Lemma 2}}{\leq}\frac{(a+b+c)^2}{3}$$

$$LHS^2\geq\frac{(a+b+c)^3}{\frac{2}{3}(a+b+c)^2}\rightarrow LHS^2\geq\frac{3}{2}(a+b+c)\rightarrow LHS\geq\sqrt{\frac{3(a+b+c)}{2}}$$

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$$\sqrt{\frac{3(a+b+c)}{2}} > \sqrt{\frac{a+b+c}{2}}$$

$$LHS = \frac{a}{\sqrt{b+c}} + \frac{b}{\sqrt{a+c}} + \frac{c}{\sqrt{a+b}} > \sqrt{\frac{a+b+c}{2}} \quad (Q.E.D)$$