If $a,b,c>0$ then:

$$\frac{a}{\sqrt{2a+3b+4c}}+\frac{b}{\sqrt{2b+3c+4a}}+\frac{c}{\sqrt{2c+3a+4b}}\ge\sqrt{\frac{a+b+c}{3}}$$

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$Lemma$ 1. (Weighted Jensen's Inequality):

Let f(x) be a convex function on an interval I. For any $x_i\in I$ and weights $w_i>0$, we have:

$$\frac{\sum w_i f(x_i)}{\sum w_i}\ge f\left(\frac{\sum(w_i x_i)}{\sum w_i}\right)$$

$Lemma$ 2. $\quad a,b,c\in R^+ \qquad (a-b)^2+(b-c)^2+(a-c)^2\ge 0$

$$2a^2+2b^2+2c^2-2(ab+ac+bc)\ge 0 \qquad a^2+b^2+c^2\ge ab+ac+bc$$

$$\begin{cases} w_1=a, w_2=b, w_3=c \\ x_1=2a+3b+4c, x_2=2b+3c+4a, x_3=2c+3a+4b \end{cases}$$

$$f(x)=\frac{1}{\sqrt{x}}\ (Decreasing\ function), \qquad \frac{d^2}{dx^2}f(x)=\frac{3}{4}x^{-\frac{5}{2}},\ \ x\in(0;+\infty), f''(x)>0$$

$$\frac{w_1f(x_1)+w_2f(x_2)+w_3f(x_3)}{w_1+w_2+w_3}\ge f\left(\frac{w_1x_1+w_2x_2+w_3x_3}{w_1+w_2+w_3}\right)$$

$$\frac{\frac{a}{\sqrt{2a+3b+4c}}+\frac{b}{\sqrt{2b+3c+4a}}+\frac{c}{\sqrt{2c+3a+4b}}}{a+b+c}$$

$$\overset{JENSEN}{\ge}\sqrt{\frac{a+b+c}{a(2a+3b+4c)+b(2b+3c+4a)+c(2c+3a+4b)}}$$

$$LHS\ge(a+b+c)\sqrt{\frac{a+b+c}{2(a^2+b^2+c^2)+7(ab+ac+bc)}}$$

$$2\sum_{cyc}a^2+7\sum_{cyc}ab\ge 3\sum_{cyc}a^2+6\sum_{cyc}ab=3(a+b+c)^2$$

$$LHS \geq (a+b+c)\sqrt{\frac{a+b+c}{3(a+b+c)^2}} = \sqrt{\frac{a+b+c}{3}}$$

$$\frac{a}{\sqrt{2a+3b+4c}} + \frac{b}{\sqrt{2b+3c+4a}} + \frac{c}{\sqrt{2c+3a+4b}} \geq \sqrt{\frac{a+b+c}{3}} \quad (Q.E.D)$$

Equality holds if and only if a = b = c