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PROBLEMS FOR JUNIORS

JP.586. If a, b, c are sides of acute ΔABC and

$$\frac{2}{a^2+c^2-b^2} = \frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2}$$

then: $\tan^2 B \geq \tan A \cdot \tan C$

Proposed by Daniel Sitaru - Romania

JP.587. If $x, y \geq 1$ then:

$$\ln(xy) \cdot (2\ln(xy)+1) \geq 4(\ln x\sqrt{\ln y} + \ln y\sqrt{\ln x})$$

Proposed by Daniel Sitaru - Romania

JP.588. If $a, b, c, d > 0, abcd = 1$ then:

$$a^{b+c+d}b^{c+d+a}c^{d+a+b}d^{a+b+c} \leq 1$$

Proposed by Marin Chirciu - Romania

JP.589. If $a, b, c \leq 0$ and $1 < \lambda \leq 2$ then:

$$\sum \frac{a(b+c)^2}{\lambda a+b+c} \leq \frac{a^2+b^2+c^2}{\lambda - 1}$$

Proposed by Marin Chirciu - Romania

JP.590. If $a, b, c > 0$ then:

$$\sum \frac{(b+c)^2}{2a^3+bc(b+c)} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

Proposed by Marin Chirciu - Romania

JP.591. If $x, y, z > 0, x+y+z = 1$ and $\lambda \geq 26$ then:

$$x^4+y^4+z^4+\lambda xyz \leq \frac{\lambda+1}{27}$$

Proposed by Marin Chirciu - Romania

JP.592. Solve for real numbers:

$$\begin{cases} 2^x+3^y+5^z=10 \\ \left|\sqrt{x^2+y^2+z^2}-\sqrt{\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}}\right|=\left|x-\frac{1}{x}\right|+\left|y-\frac{1}{y}\right|+\left|z-\frac{1}{z}\right| \end{cases}$$

Proposed by Daniel Sitaru - Romania

JP.593. Solve for real numbers:

$$\frac{x^2}{x^2+4\sqrt{x}+2}+\frac{2}{2+x\sqrt{x+2}}=\frac{4x}{5x+2}$$

Proposed by Daniel Sitaru - Romania

JP.594. Solve for real numbers:

$$\sin^{2022}x\cdot\cos^{2024}x=\frac{1}{2^{2022}}$$

Proposed by Daniel Sitaru - Romania

JP.595. Find $x,y,z>1$ such that:

$$\sum_{cyc}\frac{\log_2 x}{\log_2^6 x+\log_2^3 y+\log_2^3 z}=\frac{1}{27}\left(\sum_{cyc}\log_2 x\right)^3$$

Proposed by Daniel Sitaru - Romania

JP.596. In acute $\Delta ABC, AA', BB', CC'$ - are altitudes, $C'\in(AB), B'\in(AC), \{H\}=BB'\cap CC'$ and E,F are middle points of $[BH],[AC]$ respectively. Prove that:

$$4EF^2\geq(EC'+EB')^2+(C'F+B'F)^2$$

Proposed by Marian Ursărescu, Florică Anastase - Romania

JP.597. Let $ABCD$ be an convex quadrilateral, $\lambda\in\mathbb{R}$ and M,N be such that

$$\vec{AN}=\lambda\cdot\vec{AB}, \vec{DN}=\lambda\cdot\vec{DC}, \vec{AD}=3\cdot\vec{BC}$$

Find $\lambda\in\mathbb{R}$ such that $\vec{MN}=7\cdot\vec{BC}$

Proposed by Marian Ursărescu, Florică Anastase - Romania

JP.598. Let $n\geq 4$, and let $a_1,a_2,\ldots,a_n$ be nonnegative real numbers such that $a_1\geq a_2\geq\ldots\geq a_n$ and $a_1a_2+a_2a_3+\ldots+a_na_1=n$. Prove that:

$$\frac{1}{2a_1+5}+\frac{1}{2a_2+5}+\ldots+\frac{1}{2a_n+5}\geq\frac{n}{7}$$

Proposed by Vasile Cîrtoaje - Romania

JP.599. Prove that 3 is the largest positive value of the power k such that the inequality

$$\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}\geq a_1^k+a_2^k+\ldots+a_n^k$$

holds for $n\geq 2$ and any positive real numbers $a_1,a_2,\ldots,a_n$ with at most one $a_i<1$ and $a_1^2+a_2^2+\ldots a_n^2=n$.

Proposed by Vasile Cîrtoaje - Romania

JP.600. Calculate the limit of sequence $(a_n)_{n\geq 1}$ defined by the following relationship:

$$a_n=\frac{1}{n}\int_0^{\frac{1}{2}}\ln(1+e^{n\cdot\arcsin x})dx$$

Proposed by Vasile Mircea Popa - Romania

PROBLEMS FOR SENIORS

SP.586. Solve for real numbers:

$$\begin{cases}(\sqrt{x+y})-\sqrt{x})(\sqrt{x^2+xy}+1)=xy\\ x+y+z=3\\ (\sqrt{y+z}-\sqrt{y})(\sqrt{y^2+yz}+1)=xy\end{cases}$$

Proposed by Daniel Sitaru - Romania

SP.587. Let a,b,c be sides in ΔABC. If $\tan B=2;\tan C=3$ then:

$$a^2+b^2+c^2>\frac{2F}{3}(3\sqrt{2}+3\sqrt{5}+2\sqrt{10}-11)$$

Proposed by Daniel Sitaru - Romania

SP.588. For given $n\geq 3$, prove that 2 is the least positive value of k such that:

$$\frac{1}{ka_1+1}+\frac{1}{ka_2+1}+\ldots+\frac{1}{ka_n+1}\geq\frac{n}{k+1}$$

for any positive real numbers a_i with at most two $a_i>1$ and $a_1a_2\ldots a_n=1$

Proposed by Vasile Cîrtoaje - Romania

SP.589. Prove that 4 is the largest positive value of k such that the inequality

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq a^k+b^k+c^k$$

holds for any positive real numbers a, b, c with at most one of them less than 1 and

$$a+b=c=a^2+b^2+c^2$$

Proposed by Vasile Cîrtoaje - Romania

SP.590. Solve the following system in integers $(x,y,z)\in\mathbb{N}^*\times\mathbb{N}^*\times\mathbb{Z}$

$$\begin{cases}x^3-y^2+2z=0\\x^2+y^2+z^2=179\end{cases}$$

Proposed by Said Attaoui - Algeria

SP.591. If $a,b,c>0, a^8+b^8+c^8\leq 768$ then:

$$\sum\frac{1}{\sqrt{4+a^5}}\geq\frac{1}{2}$$

Proposed by Marin Chirciu - Romania

SP.592. If $a,b,c>0$ and $n\in\mathbb{N}, n\geq 2$ then:

$$\sum a\sqrt[n]{b^n+c^n}\geq\sqrt[n]{2}(ab+bc+va)$$

Proposed by Marin Chirciu - Romania

SP.593. Solve for real numbers:

$$(\sin x+\cos y)^2=(\sin x+1)(\cos y-1)$$

Proposed by Daniel Sitaru - Romania

SP.594. Find $x,y>0$ such that:

$$\ln^2(xy)=\ln(xe)\cdot\left(\ln\frac{y}{e}\right)$$

Proposed by Daniel Sitaru - Romania

SP.595. Solve for real numbers:

$$\tan 2x+\tan 3x+\tan 5x=\tan 2x\cdot\tan 3x\cdot\tan 5x$$

Proposed by Daniel Sitaru - Romania

SP.596. Solve for real numbers:

$$\begin{cases} \sin^2 x = \frac{1}{2} + \sin^2(y-z) \\ \sin^2 y = \frac{1}{3} + \sin^2(z-x) \\ \sin^2 z = \frac{1}{6} + \sin^2(x-y) \end{cases}$$

Proposed by Daniel Sitaru - Romania

SP.597. Let a, b, c, d be positive real numbers with $\sum a \geq \sum \frac{1}{a}$. Prove that:

$$\sum \frac{a+b+c-d}{a^4+b^4+c^4+abcd} \leq \frac{4}{3}\left(\frac{ab+ac+ad+bc+bd+cd}{abc+abd+acd+bcd}\right)$$

Proposed by Huseyin Yigit Emekci – Turkey

SP.598. Let x, y, z be positive real numbers. Prove that:

$$\frac{x^3+9xy^2}{z^3+x^2y} + \frac{y^3+9yz^2}{x^3+y^2z} + \frac{z^3+9zx^2}{y^3+z^2x} \geq 3 + \frac{12xyz(x+y+z)}{x^3y+y^3z+z^3x}$$

Proposed by Huseyin Yigit Emekci –Turkey

SP.599. We consider the function $f : D \to \mathbb{R}$

$$f(x) = x\int_x^{x+\frac{3}{x}} t \arcsin\left(\frac{1}{t}\right) dt$$

where D is the maximal domain of the function.
a. Find the domain D
b. Show that the function $f(x)$ is even
c. Calculate $\lim_{x\to-\infty} f(x)$

Proposed by Vasile Mircea Popa - Romania

SP.600. Let a, b, c, d, e, f, g be real numbers such that $a \geq b \geq c \geq d \geq e \geq f \geq g$ and $a+b+c+d+e+f+g = 0$. Prove that:

$$a^2+b^2+c^2+d^2+e^2+f^2+g^2 \geq 2(ab+bc+cd+de+ef+fg+ga)$$

Proposed by Vasile Cîrtoaje - Romania

UNDERGRADUATE PROBLEMS

UP.586. If $A \in M_{2,1}(\mathbb{R}); B \in M_{1,2}(\mathbb{R}); A \cdot B = \begin{pmatrix} 0 & 0 \\ 8 & 1 \end{pmatrix}$ then find $B \cdot A$.

Proposed by Daniel Sitaru - Romania

UP.587. Let a, b, c be positive real numbers such that at most one of them is less that 1 and $ab + bc + ca = 3$. Prove that:

$$abc(a+b+c)^3 \leq 27$$

Proposed by Vasile Cîrtoaje - Romania

UP.588. If $a \geq 0$ then:

$$15\left(\int_0^a \frac{x}{e^x}dx \cdot \int_0^a \frac{x^2}{e^x}dx \cdot \int_0^a \frac{x^3}{e^x}dx\right)^2 \leq a^9\left(\int_0^a \frac{x^2}{e^{2x}}dx\right)^3$$

Proposed by Daniel Sitaru - Romania

UP.589. If $X, Y, Z \in M_4(\mathbb{C})$ are matrices such that:

$$\begin{cases} X = 2Y + Z \\ X^2 = 4Y + 4Z \\ X^3 = 8Y + 12Z \end{cases} \quad \text{then: } X^{2024} = 2^{2024} \cdot Y + 2024 \cdot 2^{2023} \cdot Z$$

Proposed by Daniel Sitaru - Romania

UP.590. If $A, B \in M_4(\mathbb{R}); A \cdot B = B \cdot A$ then:

$$\det(A^4 + B^4 + AB(A^2 + AB + B^2)) \geq 0$$

Proposed by Daniel Sitaru - Romania

UP.591. Find:

$$\Omega = \lim_{n\to\infty} \frac{1}{3^n} \sum_{k=0}^{n-1} \sqrt{\binom{n}{k}\binom{n}{k+1}}$$

Proposed by Daniel Sitaru - Romania

UP.592. Solve for real numbers:

$$\begin{cases} \cos x + \cos y + \cos z = 1 \\ \cos^2 x + \cos^2 y + \cos^2 z = 1 \\ \cos^3 x + \cos^3 y + \cos^3 z = 1 \end{cases}$$

Proposed by Daniel Sitaru - Romania

UP.593. For $b \geq a$, prove that:

$$\int_a^b \frac{(x+1)^3 - 3x}{e^{x^3}} dx \leq b - a + \ln\left(\frac{b^3+1}{a^3+1}\right)$$

with equality if and only if $a = b$.

Proposed by Huseyin Yigit Emekci – Turkey

UP.594. Solve the system:

$$\begin{cases} x - 2y + z + 2 = k^2, \text{ with } 3 < k < 11 \\ x^2 + y^2 + z^2 = 109659 \\ -x^4 + y^2 + z^2 = 80929 \\ 3 < x < y < z, \quad x, y, z \in \mathbb{N} \end{cases}$$

Proposed by Said Attaoui – Algeria

UP.595. We consider the function $u : \mathbb{R} \to \mathbb{R}$, periodic with period 2π. For the period $[0, 2\pi]$ we have: $u(x) = 0$ if $x \in [0, \frac{\pi}{2})$; $u(x) = -\cos(x)$ if $x \in [\frac{\pi}{2}, \frac{3\pi}{2})$; $u(x) = 0$ if $x \in [\frac{3\pi}{2}, 2\pi)$. Prove the equality:

$$\int_0^\infty \frac{u(x)}{1+x^2} dx = \frac{\pi}{4e} + \frac{e^2+1}{2e} \arctan\left(\frac{1}{e}\right)$$

Proposed by Vasile Mircea Popa - Romania

UP.596. If $x > 0, y > 0, z > 0$ prove that there exists $u > 0$ such as

$$\frac{\sin x \sin y + \sin y \sin z + \sin z \sin x}{xy + yz + zx} = \frac{\sin u}{u}$$

Proposed by Cristian Miu - Romania

UP.597. Find the following limit:

$$L = \lim_{n\to\infty} \left(\frac{1}{2^n} \cdot \lim x \to \frac{\pi}{n} \left(\sum_{k=0}^{n} \binom{n}{k} \sin(k+1)x \right) \right)$$

Proposed by Marian Ursărescu, Florică Anastase - Romania

UP.598. Find the following limit:

$$L = \lim_{x\to 0} \left(\frac{1}{x} \cdot \lim_{n\to\infty} \sum_{k=1}^{n} 3^{k-1} \sin^3 \frac{x}{3^k} \right), a \in \mathbb{R}$$

Proposed by Marian Ursărescu, Florică Anastase - Romania

UP.599. Calculate the integral:

$$\int_{-\pi}^{\pi} \frac{\operatorname{arccot}(x)}{\sqrt{3-\cos(x)}} dx$$

In this problem we will consider the definition of the function $\operatorname{arccot}(x)$ which has the image the interval $(0, \pi)$.

Proposed by Vasile Mircea Popa - Romania

UP.600. If $0 < a \leq b$ then:

$$a^3 + 3\int_a^b \sinh x \cdot \operatorname{arcsinh} x dx \geq b^3$$

Proposed by Daniel Sitaru - Romania

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