ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC and $\forall$ n $\in \mathbb{N}$; n $\geq$ 2 the following relationship holds :

$$\frac{\csc^n \frac{A}{2}}{\cos \frac{A}{2}} + \frac{\csc^n \frac{B}{2}}{\cos \frac{B}{2}} + \frac{\csc^n \frac{C}{2}}{\cos \frac{C}{2}} \geq \sqrt{3}.2^{n+1}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Soumava Chakraborty-Kolkata-India

$$LHS \overset{AM-GM}{\geq} 3\sqrt[3]{\frac{\left(\prod_{cyc}\csc\frac{A}{2}\right)^n}{\prod_{cyc}\cos\frac{A}{2}}} = 3\sqrt[3]{\frac{\left(\frac{4R}{r}\right)^n.4R}{s}} \overset{\substack{Euler\\ and\\ Mitrinovic}}{\geq}$$

$$\geq 3\sqrt[3]{\frac{\left(\frac{8r}{r}\right)^n.4\left(\frac{2s}{3\sqrt{3}}\right)}{s}} = \frac{3.2^n.2}{\sqrt{3}} = \sqrt{3}.2^{n+1}$$

$\forall \Delta$ ABC,'' ='' iff Δ ABC is equilateral (QED)